1: e

If the concentration in Du Goo is x, then $\frac{10x+7(x*0.2)}{10+7} = x*11.4/17 = x*57/85 \implies \boxed{4}$

2: c

If it takes Rohit x minutes to walk a dog, then $\frac{1}{8} + \frac{1}{x} = \frac{10}{60} \implies x = 24$ so he can walk $\boxed{10}$ in 240 minutes.

3: c

If the longer diagonal has length x, then $x < 6 + 12 = 18$ from triangle inequality and the minimum length of the longest diagonal occurs when the diagonals have equal length which is when the parallelogram is a rectangle, so $x \geqslant \sqrt{6^2 + 12^2} \approx 13.4$, so $x \in \{14, 15, 16, 17\} \implies \boxed{62}$

4: a

Treating OO and ER as single units, there are 4! ways to order the word. Multiply by 2! ways to order the E and R is 48. Subtract 1 to not spell booger so $\boxed{47}$

5: c

The fourth row is $13, 21, 34, 55 \implies \boxed{123}$

6: b

The vertical distance can be found by subtracting the equations to get $x^2 - 2x + 5$, which is minimized at $x = \frac{-(-2)}{2} = 1$ and returns 4. The horizontal distance is $|k|$ if the equation $y = (x - k)^2 + 4$, which is the parabola translated k units horizontally, is tangent to the equation $y = 2x - 1$. Setting equal we have $x^2 - x(2k + 2) + k^2 + 5$. We want this to have only one solution for the equations to be tangent, so $b^2 - 4ac = 0 \iff (-2k - 2)^2 = 20 +$

$4k^2 \implies 4k^2+8k+4 = 4k^2+20 \implies k=2$ and the sum is $\boxed{6}$

7: b

$\log(a) = a\log\left(\frac{1}{\sqrt{2}}\right) \implies \frac{\log(a)}{a} = \boxed{-\frac{1}{2}\log 2}$

8: d

$1+2+3+4+6+7+8+9+11+22+33+44+55+66+77+88+99+101+111+121+131 = 999 \implies \boxed{21}$

9: c

The tangent segments to each excircle intersecting at the vertices of the triangle are of equal length of 1/2, so the 30-60-90 triangle formed from the perpendicular from one of the excenters to either of the extended sides it is tangent to has longer leg of length 3/2 and hypotenuse of length $\sqrt{3}$, meaning the exradii are of length $\frac{\sqrt{3}}{2}$ and the distance from the excenter to the center of ABC is $\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{6}=\frac{2}{\sqrt{3}} \implies \boxed{\frac{4\pi}{\sqrt{3}}}$

10: a

Taking the sum of the first equation, 3 times the second, 1/2 times the third, and the fourth, we have $a^3+3a^2b+3ab^2+b^3 = -8+57+15 \implies (a+b)^3 = 64 \implies a+b = \boxed{4}$

11: 153

We wish to minimize the maximum value out of the following times it takes for each friend to reach the restaurant: $\frac{|x|}{3}, \frac{|x-2|}{4}, \frac{|x-5|}{5}, \frac{|x-9|}{6}, \frac{|x-12|}{7}$, where x is the distance from 0 of the restaurant. The candidates for longest time to reach the restaurant, if the restaurant is between

Berek's and Derek's houses, are Aerek and Eerek. This is because $\frac{|x-12|}{7} > \frac{|x-5|}{5}, \frac{|x-9|}{6}$ on the interval from 0 to Derek's house, and $\frac{|x|}{3} > \frac{|x-2|}{4}, \frac{|x-5|}{5}$ on the interval from Berek's house onwards. We then narrow our minimization attempts to Aerek's time and Eerek's time. The longer of the two times is minimized when they are equal, because decreasing one would increase the other. Then, $\frac{|x|}{3} = \frac{|x-12|}{7} \implies x = \frac{18}{5} \implies \boxed{153}$

12: 556

We can "unwrap" the string around the pole to get a right triangle with height 36 inches and base length $8*2\pi$ inches, meaning the length of the string is the length of the hypotenuse which is $\sqrt{1296+256\pi^2}$. The area is then $1296\pi + 256\pi^3 \implies 1\boxed{556}$

13: 018

The possible sequences leading to a loss with c meaning Alex clears and n meaning he doesn't are $nnnnnn$, $cnnnnnn$, $ccnnnnnn$, $ncnnnnnn$. The probabilities are $\left(\frac{8}{9}\right)^6 + \left(\frac{1}{9}\right)\left(\frac{8}{9}\right)^6 + \left(\frac{1}{9}\right)^2\left(\frac{8}{9}\right)^6 + \left(\frac{8}{9}\right)\left(\frac{1}{9}\right)\left(\frac{8}{9}\right)^6 = \frac{2^{18}*99}{3^{16}} = \frac{2^{18}*11}{3^{14}} \implies |e_1e_2e_3p_1p_2p_3| = 16632 \implies \boxed{018}$

14: 224

From the given radii, we see that the intersections between the inner circle and vertical axis connect with the center of the inner circle to form a 90 degree sector. Then the part of the inner circle on the left of the vertical axis is $\frac{\pi r^2}{4} - \frac{r^2}{2}$. Subtract this from half the area of the outer circle which is $\frac{1}{2}\left(\frac{2}{3}\right)^2\pi$ to get $\frac{2\pi}{9} - \frac{\pi r^2}{4} + \frac{r^2}{2}$. Set this equal to the area of the hexagon in terms of s to get $\frac{3\sqrt{3}s^2}{2} =$

$\frac{2\pi}{9} - \frac{\pi r^2}{4} + \frac{r^2}{2}$. Solving and plugging in $r = \frac{2}{3}(2 - \sqrt{2})$ yields $s^2 = \pi\left(\frac{8}{81}\left(\sqrt{6} - \sqrt{3}\right)\right) + \frac{2}{3}\sqrt{3} - \frac{4}{9}\sqrt{6} \implies \boxed{224}$

15: 448

Recognize $\frac{1}{1-r}$ as the infinite geometric series $1 + r + r^2 + \dots$, and we see that we want $r = \frac{6}{7}$ to get $f(r) = 7$, so we plug in $\frac{6}{7}$ into $1+r+r^2+r^3$ which is up to the third degree term and this gives us $\frac{1105}{343} \implies 1\boxed{448}$