The Excel Math Competition



Date: 3/9/2025

- 1. This is a 45 minute individual exam.
- 2. No collaboration is allowed.
- 3. The first 10 questions are worth [5] points each and are multiple choice.
- 4. The last 5 questions are worth [10] points each and are short response.
- 5. Each of the final 5 questions have answers which are positive integers between 000 and 999, inclusive.
- 6. The questions are arranged in roughly ascending difficulty.
- 7. If you believe a question is seriously flawed, or have an answer which is not one of the listed answers, there will be a 10-minute dispute period after the test, after which no disputes will be accepted.
- 8. In the event of a dispute, **leave the question blank** and submit a dispute form after the test.
- 9. All disputes will be considered on an individual-by-individual basis, so no student will not receive credit if they did not submit a dispute, except for in the case of a question being thrown out.
- 10. There are (intentionally) quite a few tricky questions on this test! Answer as many as you can, and have fun!

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$1 \quad [5]$

Siyuan has 10 gallons of Du Goo^{TM} . Geofferey, on the other hand, prefers his patented Yu Goo^{TM} , which has 20% the concentration of \mathfrak{DU} , in parts per million (ppm), compared to Du Goo^{TM} . Geoff likes Yu Goo^{TM} so much that he has consumed all but 7 gallons of his supply. Fearing their supplies will get confiscated, the two combine their remaining rations. This new mixture has a concentration of \mathfrak{DU} which is a fraction $\frac{a}{b}$, in simplified form, of Du Goo^{TM} 's initial \mathfrak{DU} concentration. What is the tens digit of $a \cdot b$?

a) 0 b) 1 c) 2 d) 3 e) 4

$2 \quad [5]$

Siyuan has a job walking dogs, and he's the most efficient dog-walker around, being able to walk 1 dog every 8 minutes. He and Rohit decide to team up to make more money, and together they can walk 10 dogs per hour. However, they had a falling out and decide not to work together anymore. Rohit spends the next 4 hours walking dogs alone. How many dogs does he walk?

a) 6 b) 8 c) 10 d) 12 e) 14

3 [5]

A parallelogram has sides of length 6 and 12. What is the sum of the possible integer lengths of the longer diagonal of this parallelogram?

a) 58 b) 60 c) 62 d) 64 e) 66

$4 \quad [5]$

Bhav HATES the nickname that Rodion gave him, and he'd do anything to change it, including permuting the letters to a different arrangement. How many ways are there to arrange the letters in the word \mathbb{BOOGER} so as to not spell \mathbb{BOOGER} , and so that the two O's are adjacent, and the E and R are adjacent?

5 [5]

Pascal (no relation) decides to write some numbers in a triangle shape (no relation). The rows are filled in from left to right with the *Fibonacci* numbers in order. Each *Fibonacci* number is defined as the sum of the two previous *Fibonacci* numbers with the first two being 1 and 1, so that the first row has one 1, and the second row has a 1 and a 2. What is the sum of the entries in the 4'th row of Pascal's triangle?

a) 93 b) 120 c) 123 d) 140 e) 143

$6 \quad [5]$

Find the sum of the shortest **vertical** distance and shortest **horizontal** distance in the xy-plane between the two curves

$$y = 2x - 1$$
$$y = x^2 + 4$$

on the interval [-5, 5]

a) 4 b) 6 c)
$$\frac{15}{2}$$
 d) $2 + \sqrt{37}$ e) 12

 $7 \quad [5]$

If
$$a = \left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{1}{\sqrt{2}}\right)^{\cdots}}}$$
 and $f(x) = \frac{\log x}{x}$, find $f(a)$.
a) $-\log 2$ b) $-\frac{1}{2}\log 2$ c) 1 d) $\frac{1}{2}\log 2$ e) $\log 2$

8 [5]

What is the maximum number of **distinct** palindromes which can be summed to equal a 3-digit palindrome? Recall a palindrome is a positive integer which is written the same both forward and backward.

a) 18 b) 19 c) 20 d) 21 e) 22





An equilateral triangle ABC of side length 1 has all of its sides extended, and three circles drawn tangent to each of the extended sides and a third side, as shown. These are called the *excircles* of ABC. A unique circle passes through the centers of all three excircles. What is the perimeter of this circle?

a)
$$\frac{\pi}{\sqrt{3}}$$
 b) $\frac{2\pi}{\sqrt{3}}$ c) $\frac{4\pi}{\sqrt{3}}$ d) $\frac{6\pi}{\sqrt{3}}$ e) $\frac{8\pi}{\sqrt{3}}$

10 [5]

Find the real value of a + b for the following system of equations:

$$a^{3} + ab = -8$$

$$ab^{2} + 5c = 19$$

$$6a^{2}b - 2ab + 4d = 30$$

$$b^{3} = 2d + 15c$$

a) 4 b) $\frac{8}{3}$ c) 5 d) $\sqrt{10}$ e) 12

11 [10]

Five friends Aerek, Berek, Cerek, Derek, and Eerek live along Bifty-Nifty Road at mile markers 0, 2, 5, 9, and 12 respectively. They want to meet at a restaurant to steal all of the food there but there is a small problem. Berek and Derek always drive at outrageous speeds and tend to end up in trouble. So, the group of friends decide on going to a place within walkable distance. Their walking speeds are 3, 4, 5, 6, and 7 mph, respectively. They all leave from their homes at the same time and then walk directly toward the restaurant. The friends wish to choose the restaurant's location so that the longest time any one of them must walk is as short as possible. The distance from 0, in miles, of the restaurant that would achieve this can be represented as a simplified fraction $\frac{a}{b}$. What is a + b + 2ab - 50?

$12 \quad [10]$



Alex Oahz is playing tetherball with Sam Agetro. The setup for tetherball is a ball attached to a string, which both players try to hit so as to wrap the string 8 times around the 3 foot high pole of diameter 2 inches as shown. One end of the string is attached to a point on the circumference of the top face of the pole. At present, the ball is traveling very fast in a perfectly horizontal circular trajectory (which is centered at the point the string attaches to the pole) and is not wrapped around the pole at all. The area of this circular path can be written as $a\pi^b + c\pi^d$. What is the remainder when a + b + c + d is divided by 1000.

13 [10]

Alex grows tired of beating Sam in tetherball, so he opens up his iPad and starts playing *Block Blast*. Alex is not very good at *Block Blast*, so on any given move he has a tragic $\frac{1}{9}$ chance of clearing at least one file (a file is a row or column). If he goes 6 moves without clearing **at least one** row or column, he will lose. The probability that Alex loses within 8 moves of a new game can be written as $p_1^{e_1}p_2^{e_2}p_3^{e_3}$ for distinct primes p_i . What is the sum of the digits of $|e_1e_2e_3p_1p_2p_3|$?



Forrest has an oddly shaped shrimp. It has a large outer radius of $\frac{2}{3}$ inches, and a smaller inner radius of $\frac{2}{3}(2-\sqrt{2})$ inches, as shown. The outer and inner circles are internally tangent at a point and centered at the origin. Forrest decides to process the shrimp into a regular hexagonal shrimp cracker, but first he must remove the head and tail, by discarding the area on the right of the vertical axis shown. The square of the side length of this cracker can be written as $\pi \left(\frac{a^3}{b^4}\left(\sqrt{c}-\sqrt{d}\right)\right) + \frac{a}{b}\sqrt{d} - \frac{a^2}{b^2}\sqrt{c}$ for some squarefree, not necessarily distinct positive integers a, b, c, d. Find $a^b + c^d$

15 [10]

In calculus, you will learn about *polynomial approximations* to functions. These are polynomials which approximate a non-polynomial function arbitrarily well as you take higher and higher degree terms (this is a bit of a simplification). Approximate the value of 7 using the polynomial approximation of the function $f(r) = \frac{1}{1-r}$ up to the third degree term. The value you will get is a fraction $\frac{a}{b}$ in lowest terms. What is the remainder when a + b is divided by 1000? Note: you don't need any calculus for this question. Also, I know it seems silly to "approximate the value of 7," but it is a useful exercise nonetheless.