1

the semimajor axis a is evidently of length $\frac{152+147}{2} = 149.5$ million kilometers, and so the focal length c is then $152 - 149.5 = 2.5$ million kilometers, so $e = \frac{c}{a} \approx \frac{2.5}{149.5} \approx \boxed{\frac{1}{60}}$

2

if $D = (0,0)$ and $K = (1,0)$, then $R = (\frac{1}{2}, \frac{\sqrt{3}}{2})$, $M = (\frac{1}{2}, \frac{\sqrt{3}}{6})$, $P = (\frac{3}{2}, \frac{7\sqrt{3}}{6})$, $L = (\frac{5}{2}, \frac{\sqrt{3}}{6})$, and by shoelace theorem the area is $\boxed{\frac{3\sqrt{3}}{2}}$

3

We use complementary counting. Each of them must use up a slot out of the 5 available in the first round of choices, leaving 2 un-chosen ideas. Now we work on the cases where 1) they all succeed on the first try, 2) one of them must re-pick once, and 3) either one of them re-picks twice or two of them must re-pick. Case 1 occurs with probability 1/8, Case 2 occurs with probability (3)(1/8)(1/2), and Case 3 occurs with probability (3)(1/8)(1/2)(1/2)+(3)(1/8)(1/2)(1/2). These all add to 1/2, whose complement is also $\boxed{\frac{1}{2}}$

4

They all bench at least one time, so they bench a total of at least 16 lbs. Then, any number of additional lbs benched by Alex could also be benched by Siyuan in double the number of lifts, as $6 = 3 * 2$. Thus, by chicken mcnugget theorem on 7, 3, we have that the maximum unattainable number of lbs benched is $16 + (7 * 3 - (7 + 3)) = \boxed{27}$

5

After giving out n pepperoni and $2n$ cheese pizzas, there are $10-n$ pepperoni and $30-2n$ cheese remaining and a total of $40 - 3n$. Exactly $\frac{1}{n}$'th of the total remaining pizzas are pepperoni because Grister Migelis can split them evenly to distribute. $\frac{1}{n}(40 - 3n) = 10 - n \implies n^2 - 13n + 40 = 0 \implies n \in \{5, 8\} \implies \boxed{13}$

6

there are a total of $\frac{4\pi R^2}{R^2} = 4\pi$ steradians in a full sphere. The proportion of which are land on the earths surface is 0.29. Thus, $k = 0.29 \cdot 4 = 1.16 \implies \boxed{11}$

7

Notice that $(r+s+t)(\frac{1}{r} + \frac{1}{s} + \frac{1}{t}) = 3 + \frac{r+s}{t} + \frac{s+t}{r} + \frac{r+t}{s} = -(-2025)(-\frac{(-4)}{1}) = 8100$ by Vieta's theorems. Thus the answer is 8097 so $\boxed{24}$

8

draw 6 points on the circumference of Ω, and we want to connect 3 distinct pairs. Notice there is only one distinct way to satisfy the given conditions (or 0 in cases with infinitesimal probability like the points being evenly spaced, but we ignore these), when each chord separates the circumference into 2 parts each with 2 points. The total number of ways to draw these connections are $6-1=5$ for the first pair once we fix a starting point, and 3 for the other 2 pairs (they can be parallel 2 ways or intersecting), and so we get $\boxed{\frac{1}{15}}$

9

Notice that $f(1)-f(-1)=(a_0+a_1x+a_2x^2+\dots)-(a_0-a_1x+a_2x^2\mp\dots)=$ two times the requested sum. Thus, $\frac{f(1)-f(-1)}{2}=\frac{-\frac{5}{3}+1}{2}=\boxed{-\frac{1}{3}}$

10

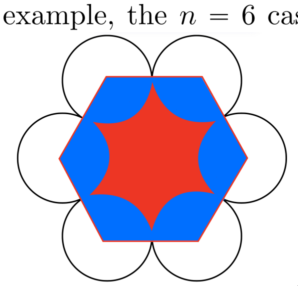
$\lfloor x\rfloor+\lceil x\rceil-x=x-2\{x\}+1$, which when sketched looks like a bunch of segments of x-width 1 and slope -1 following the line $y=x$. The area under these segments can be easily calculated using integer intervals: from x=0 to 1 the area is 1/2. From 1 to 2 it is 3/2. From 2 to 3 it is 5/2, and etc. to get $\boxed{18}$

11

37 divides a string of 1's iff the string is of length divisible by 3, as 111=3*37. Note that $255=11111111_2$ is 8 digits (and the maximum binary value possible with 8 digits). Thus, we want to find the number of permutations of 8 total 0's and 1's such that there are either 3 or 6 1's. This is $\binom{8}{3}+\binom{8}{6}=56+28=\boxed{84}$

12

We first find a general term for A_n. The requested area is the difference between the area of the regular n-gon with vertices at the centers of the n circles and side length 2, and the summed area of the sectors of the n circles which are bounded by the segments connecting their points of tangency and centers. For example, the $n=6$ case looks like the difference between the red hexagon and blue sectors:



. We have that the area of the polygon is its inradius times its semiperimeter. The inradius is $1\cdot\tan\frac{1}{2}\theta$, where $\theta=\frac{\pi(n-2)}{n}$ is the internal angle of the polygon. The semiperimeter is 1/2 times the perimeter which is $2n$ so the area is $n\tan\left(\frac{\pi(n-1)}{2n}\right)$. Each of the circular sectors has internal angle θ and contributes an area of $\frac{\theta}{2\pi} \cdot \pi(1^2) = \frac{\theta}{2}$ for a total area of $\frac{\pi(n-2)}{2}$. Thus, $A_n = n\tan\left(\frac{\pi(n-1)}{2n}\right) - \frac{\pi(n-2)}{2}$. For any n, the value of $\frac{c}{ab} = \frac{\frac{\pi(n-2)}{2}}{\frac{\pi(n-2)}{2n}n} = \boxed{1}$

13

We have (for $n \geqslant 3$) that $\frac{S_n - S_{n-1}}{S_{n-1} - S_{n-2}} = r = 3$ by definition, so $S_n = 4S_{n-1} - 3S_{n-2}$. Hence, $15 = 4S_2 - 3S_1 \implies S_2 = \frac{3}{4}S_1 + \frac{15}{4}$. To be strictly increasing $15 > S_2 > S_1$. Letting $S_2 = y, S_1 = x$ and graphing, this agrees with the previous equation when $S_1 \in (0, 15)$. Finally, to be all integers, we just need $S_1, S_2 \in \mathbb{Z}$, as r is an integer. Thus, we want $3S_1 + 15 \equiv 0 \pmod 4 \implies S_1 \equiv 3 \pmod 4$, which narrows our choices to $S_1 \in \{3, 7, 11\} \implies \boxed{21}$

14

Notice that from its definition, $\mu(p_1^{e_1}p_2^{e_2}\dots) = 0$ when any of $e_i > 1$, so we only need to consider x of the form $\prod p_i$ and 1. The primes at most 15 are $p_i = \{2, 3, 5, 7, 11, 13\}$. For x with 0 distinct prime divisors, i.e. $x = 1$, we have $x\mu(x) = 1$. For x with 1 distinct prime divisor, we have $\sum x\mu(x) = (-1)(\sum x) = -\sum p_i$. For x with 2 distinct prime divisors, we have $\sum x\mu(x) = \sum p_i p_{j\neq i}$, and in general we see that for x with a distinct prime factors, the contribution to $\sum x\mu(x)$ is $(-1)^a$ times the a'th elementary symmetric sum $\sum_{sym}^{a} p_i$ of elements in p_i. We recognize this signed sum as the binomial expansion of $\prod(1 - p_i)$. Thus, the desired sum is just $(1-2)(1-3)(1-5)(1-7)(1-11)(1-13) = \boxed{576}0$

15

$\tan(xy) = 1 \implies xy = \frac{\pi}{4} + \pi n$ for integer n. Thus, the graph of $\tan(xy) = 1$ looks like the union of the graphs of all hyperbolas of the form $xy = \frac{\pi}{4} + \pi n$. Now, the closest points to the origin on each of these hyperbolas are the only viable points of tangency for a circle centered at the origin. As each hyperbola is tilted at 45 degrees, we find the intersection of each with either $y = x$ for nonnegative n, or $y = -x$ for negative n, and find the distances r_n from these points to the origin. $x^2 = \frac{\pi}{4} + \pi n \implies r_n = \sqrt{2}\sqrt{\frac{\pi}{4}}, \sqrt{2}\sqrt{\frac{5\pi}{4}}, \sqrt{2}\sqrt{\frac{9\pi}{4}}, \dots$, and $-x^2 = \frac{\pi}{4} + \pi n \implies r_n = \sqrt{2}\sqrt{\frac{3\pi}{4}}, \sqrt{2}\sqrt{\frac{7\pi}{4}}, \sqrt{2}\sqrt{\frac{11\pi}{4}}, \dots$, so that all r_n are given by $\sqrt{\frac{\pi(2n+1)}{2}}$ for **nonnegative** n. Thus, the requested sum is now (and using the hint)

$$\sum_{n=0}^{\infty} \frac{4}{\pi^4(2n+1)^2} = \frac{4}{\pi^4} \cdot \frac{\pi^2}{8} = \frac{1}{2\pi^2} \implies \boxed{4}$$