The Excel Math Competition

May 2025(Advanced Algebra/Geometry Exam)

1. This is a 60 minute individual exam.
2. No collaboration or external devices (like a calculator) are are allowed.
3. The first 10 questions are worth [5] points each and are multiple choice.
4. The last 5 questions are worth [10] points each and are short response.
5. Each of the final 5 questions have answers which are positive integers between 000 and 999, inclusive.
6. The questions are arranged in roughly ascending difficulty.
7. If you believe a question is seriously flawed, or have an answer which is not one of the listed answers, there will be a 10-minute dispute period after the test, after which no disputes will be accepted.
8. In the event of a dispute, **leave the question blank** and let your proctor know after the testing time ends.
9. All disputes will be considered on an individual-by-individual basis, so no student will receive credit if they did not submit a dispute, except for in the case of a question being thrown out.

Test writer:
Joey Levenston

Test Editor:
David Zapata

Problem contributors:
Joey Levenston
Ryan Rohit
Alex Zhao
Rayan Kha

Sponsors:

WOLFRAM

Jane Street

AoPS

1 [5]

Welcome (or welcome back) to the ExcelAcademe Math Competition! As the last full month of spring, everyone's hopefully looking forward to a nice summer break! During summer, Earth is at the farthest point from the Sun in its elliptical orbit, at about 152 million kilometers away. The time when Earth is closest to the Sun (which is at the focus of its orbit) is in winter, at about 147 million kilometers away. Which of the following is closest to the eccentricity of the Earth's orbit around the sun?

a) $\frac{1}{120}$ b) $\frac{1}{60}$ c) $\frac{1}{45}$ d) $\frac{1}{30}$ e) $\frac{1}{15}$

2 [5]

Consider equilateral $\triangle RAL$ with side length 1 and equilateral $\triangle DMP$ with parallel sides to $\triangle RAL$, D at the center of $\triangle RAL$, and side length 2. Find the area of convex pentagon $RAMPL$.

a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{3}$ c) $\frac{3\sqrt{3}}{2}$ d) $2\sqrt{3}$ e) $\frac{5\sqrt{3}}{2}$

3 [5]

David, Michael, and Brendan are scrambling to finish their engineering projects! Each of them has to choose a distinct project idea out of a pool of 5 ideas. However, they all have a 50% chance of completely failing to complete any given project and having to pick another un-chosen idea from the pool of remaining ideas. What is the probability that at least one of the three engineers runs out of possible project ideas before they all complete a project?

a) $\frac{1}{6}$ b) $\frac{1}{5}$ c) $\frac{1}{4}$ d) $\frac{1}{3}$ e) $\frac{1}{2}$

4 [5]

Zohar can bench 7 lbs. Alex can bench 6 lbs. Siyuan can bench 3 lbs. What is the maximum number of lbs they *cannot* bench in total, if they all bench a positive number of times without tiring?

a) 20 b) 21 c) 24 d) 27 e) 31

5 [5]

This MAO national convention, a certain school has 40 attendees. Grister Migelis knows that 25% of these attendees ordered a pepperoni pizza, and the rest ordered a cheese pizza. However, some of the students are very greedy and say they ordered pepperoni while having ordered only cheese! He doesn't notice this until he has already given out n pepperoni pizzas and $2n$ cheese pizzas. Trying to salvage the situation, Grister Migelis compromises with the remaining students by mixing and matching slices and giving each of them a pizza which is $\frac{1}{n}$'th pepperoni and the rest cheese. He manages to do this with all of the remaining students successfully. What is the sum of the possible values of n?

a) 5 b) 8 c) 11 d) 13 e) 15

6 [5]

A *steradian* is a unit (denoted Ω) used to measure the *solid angle* in 3-D space, and is defined as the ratio between the surface area covered on a sphere and R^2, the squared radius of that sphere. The earth's surface is about 71% water. The steradian measure of the earth which is covered by **land** can be written as $k\pi$. Find $\lfloor 10k \rfloor$.

a) 11 b) 14 c) 17 d) 20 e) 23

7 [5]

Consider $f(x) = x^3 - 2025x^2 - 4x + 1$ with roots r, s, t. Find the sum of the digits of $\frac{r+s}{t} + \frac{s+t}{r} + \frac{r+t}{s}$.

a) 20 b) 24 c) 27 d) 30 e) 36

8 [5]

Three chords are independently drawn at random in circle Ω. What is the probability that they form a non-degenerate triangle entirely contained within Ω?

a) $\frac{1}{18}$ b) $\frac{1}{15}$ c) $\frac{1}{7}$ d) $\frac{1}{5}$ e) $\frac{1}{3}$

9 [5]

There exist real numbers a_n with $n \geqslant 0$ such that

$$f(x) = \frac{2x^2+3}{x-4} = \sum_{n=0}^{\infty} a_n x^n$$

For $|x| < 4$. For the purposes of this question you may assume this polynomial is simply an equivalent way to represent the function f on this interval. Determine

$$\sum_{n=0}^{\infty} a_{2n+1}$$

a) -1 b) $-\frac{2}{3}$ c) $-\frac{1}{3}$ d) $\frac{1}{3}$ e) $\frac{2}{3}$

10 [5]

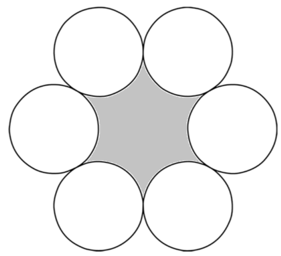
Find the area between the x-axis and the graph of $f(x) = \lfloor x \rfloor + \lceil x \rceil - x$ from $x = 0$ to $x = 6$.

a) 6 b) 9.5 c) 12 d) 15.5 e) 18

11 [10]

Let $f(x)$ be the concatenation of every 1 in the base 2 representation of k. For example, $f(5) = 11$ as $5 = 101_2$. Find the number of positive integers $n < 256$ satisfying $37 \mid f(n)$.

12 [10]



n circles of radius 1 are each pairwise externally tangent to 2 of the others and arranged such that their centers lie on a larger common circle. As an example, the $n = 6$ case is provided. A_n is defined for $n \geqslant 3$ as the area bounded by the inner-facing portions of the circles, as shown above in the shaded region for A_6. Given that A_{2025} can be written as $a \tan(b\pi) - c\pi$ for real numbers a, b, c, determine $\frac{c}{ab}$.

13 [10]

Satvik has been gifted a certain sequence $S_n = \{s_1, s_2, \dots\}$ for his birthday. S_n is special in that the differences between the $(n+1)$'th and n'th terms for $n \geqslant 1$ form a geometric sequence with common ratio r. Satvik knows the following about his sequence: $r = 3$, $s_3 = 15$, and S_n consists of strictly increasing, positive, integer terms. Help Satvik find the sum of possible values of s_1.

14 [10]

We define the *Möbius function* $\mu(n)$ on a positive integer n as the following:

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1 \\ (-1)^k & \text{if } n \text{ is the product of } k \text{ distinct primes} \\ 0 & \text{if } n \text{ is divisible by a perfect square greater than 1} \end{cases}$$

If S is the set of all positive integer divisors of $15!$, determine

$$\frac{1}{10} \sum_{x \in S} x \cdot \mu(x)$$

15 [10]

Call Ω the set of distinct circles centered at the origin which are tangent to the graph of $\tan(xy) = 1$ at some point. Note: these circles may intersect the graph at other points.

Call $A(\omega)$ for a circle $\omega \in \Omega$ the area of that circle. The following sum can be written as $\frac{1}{a}\pi^b$ for integers a, b. Find $a - b$

$$\sum_{\omega \in \Omega} \frac{1}{(A(\omega))^2}$$

Hint! $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots = \dfrac{\pi^2}{8}$